

Recurrent Neural Network Training with Dark Knowledge Transfer

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Abstract

Recurrent neural networks (RNNs) are powerful in modeling sequential data such as speech signals and hence are suitable for acoustic modeling in automatic speech recognition (ASR). Although some successful stories have been reported, training RNNs remains highly challenging. Recent research found that a well-trained model can be used as a source of ‘dark knowledge’ to train other models, by use of soft targets generated by the teacher model. This idea has been employed to train simple models from powerful yet complex models. In this paper, we perform an opposite research that uses a simple model to train a complex model such as an RNN. We show that the knowledge transfer learning with soft targets leads to a smoother objective function which simplifies complex model training and improves model generalizability. Additionally, the knowledge transfer learning can be combined with the traditional supervised learning, in a way of either regularization or pre-training. Experiments on an ASR task demonstrated that this method can be successfully applied to train RNNs, without any tricks on the learning scheme (such as momentum) or Hessian approximation.

Keywords: speech recognition; deep neural network; dark knowledge; recurrent neural network

1 Introduction

Deep learning has gained significant success in a wide range of applications, for example, automatic speech recognition (ASR) [1, 2, 3, 4, 5, 6]. A powerful deep learning model that has been reported effective in ASR is the recurrent neural network (RNN)[7, 8, 9, 10, 11, 12]. An obvious advantage of RNNs compared to conventional deep neural networks (DNN) is that RNNs can model long-term dynamic properties of speech signals.

A simple training method for RNNs is the back propagation through time algorithm [13]. This first-order approach, however, is rather inefficient due to two major difficulties in modeling long-term temporal dependency: (1) the twists of the objective function caused by the high nonlinearity; (2) the vanishing and explosion of gradients in back propagation [14]. In order to address these difficulties (mainly the second), a modified architecture called the long short-term memory (LSTM) was proposed in [15] and has been successfully applied to ASR [16]. In the echo-state-network (ESN) architecture proposed by [17], the hidden-to-hidden weights are not learned in the training so the problem of odd gradients does not exist. Recently, a special variant of the Hessian-free (HF) optimization approach was successfully applied to learn RNNs from random initialization [18, 19]. A particular problem of the HF approach is that the computation is demanding. Another recent study

shows that a carefully designed momentum setting can significantly improve RNN learning, with limited computation and can reach the performance obtained with the HF method [20]. Although these methods can address the difficulties of RNN training to some extent, they are either too tricky (e.g., the momentum method) or less optimal (e.g., the ESN method).

In this paper, we present a simple yet powerful approach to learn RNNs by knowledge transfer. This work is largely motivated by the recently proposed logit matching [21] and dark knowledge distiller [22]. The basic idea of the knowledge transfer approach is that a well-trained model involves rich knowledge of the target task and can be used to guide training other models. Current research focuses on learning simple models (in terms of structure) from a powerful yet complex model, or an ensemble of models [21, 22]. In ASR, this idea has been employed to train small DNN models from a large and complex one [23].

In this paper, we conduct an opposite study, which employs transfer learning to train complex models guided by a simple model. Different from the existing research that tries to distill knowledge from the teacher model, we treat the teacher model as a regularization so that the objective function of the complex model is smoothed, or a pre-training step so that the supervised training can be located at a good starting point. This in fact leads to a new training approach that is easy to perform and can be extended to any model architecture. We employ this idea to address the difficulties in RNN training. The experiments on an ASR task with the Aurora4 database verified that the proposed method can significantly improve RNN training.

The rest of the paper is organized as follows. Section 2 briefly discusses some related works, and 3 presents the methods. Section 4 presents the experiments, and the paper is concluded by Section 5.

2 Related to prior work

This study is directly motivated by the work of dark knowledge distillation [22]. The important aspect that distinguishes our work from others is that the existing methods focus on distilling knowledge of complex model and use it to empower simple models, whereas our study uses simple models to teach complex models. The teacher model in our work in fact knows not so much, but it is sufficient to provide a rough guide that is important to train complex models, such as RNNs in the present study. More precisely, the existing methods use the teacher model as a knowledge source to learn from, while our method uses the teacher model as a regularization or for pre-training.

Another more related work is the knowledge transfer between DNNs and RNNs, as proposed in [24]. However, it employs knowledge transfer to train DNNs with RNNs. This still follows the conventional idea described above, and so is different from ours, even though both work on the same deep learning models and employ the same technique.

3 RNN training with knowledge transfer

3.1 Dark knowledge distiller

The idea that a well-trained DNN model can be used as a teacher to guide the training of other models was proposed by several authors almost at the same

time [21, 22, 23]. The basic assumption is that the teacher model encodes rich knowledge for the task in hand and this knowledge can be distilled to boost the child model which is often simpler and can not learn many details without the teacher’s guide. There are a few ways to distill the knowledge. The logit matching approach proposed by [21] teaches a child model by encouraging its logits (activations before softmax) close to those of the teacher model in terms of the ℓ_2 norm, and the dark knowledge distiller model proposed by [22] encourages the posterior probabilities (softmax output) of the child model close to those of the teacher model in terms of cross entropy. This transfer learning has been applied to learn simple models to approach the performance of a complex model or a large model ensemble, for example, learning a small DNN from a large DNN [23] or a DNN from a more complex RNN [24].

We focus on the dark knowledge distiller approach as it showed better performance in our experiments. Basically, a well-trained DNN model plays the role of a teacher and generates posterior probabilities of the training samples as new targets for training other models. These posterior probabilities are called ‘soft targets’ since the class identities are not as deterministic as the original one-hot ‘hard targets’. To make the targets softer, a temperature T can be applied to scale the logits in the softmax, formulated as $p_i = \frac{e^{z_i/T}}{\sum_j e^{z_j/T}}$ where i, j index the output units. The introduction of T allows more information of non-targets to be distilled. For example, a training sample with the hard target $[1 \ 0 \ 0]$ does not involve any rank information for the second and third class; with the soft targets, e.g., $[0.8, 0.15, 0.5]$, the rank information of the second and third class is reflected. Additionally, with a large T applied, the target is even softer, e.g., $[0.6, 0.25, 0.15]$, which allows the non-target classes to be more prominent in the training. Note that the additional rank information on the non-target classes is not available in the original target, but is distilled from the teacher model. Additionally, a larger T boosts information of non-target classes but at the same time reduces information of target classes. If T is very large, the soft target falls back to a uniform distribution and is not informative any more. Therefore, T controls how the knowledge is distilled from the teacher model and hence needs to be set appropriately according to the task in hand.

3.2 Dark knowledge for complex model training

Dark knowledge, in the form of soft targets, can be used not only for boosting simple models, but also for training complex models. The basic assumption is that soft targets lead to a smoother objective function that is easier to optimize compared to the original one. Intuitively, soft targets offer probabilistic class labels which are not as arbitrary as hard targets. This matches the real situation where uncertainty always exists in classification tasks. For example, in speech recognition, it is often difficult to identify the phone class of a frame due to the effect of co-articulation. Moreover, the uncertainty associated with soft targets blurs the decision boundary of correct and incorrect classes. The smoothness associated with soft targets has been noticed in [22], which states that soft targets result in less variance in the gradient between training samples. This is equal to say that the objective function is less twisted, or smoother in another word. A smooth objective function is certainly

much easier to optimize, and in the extreme case where the targets are very soft (i.e., T goes to infinity), the objective function becomes flat and the optimization is trivial.

In order to investigate the smoothness of the objective function with soft targets, we study a simple neural network, the logistic regression model formulated as $y = \frac{1}{1+e^{-w^T x}}$ where $x \in R^2$ and $y \in R$ represent the input and output variables respectively, and $w \in R^2$ represents the model parameters. A number of samples $\{x_i\}$ are sampled from two classes, following the two Gaussian mixture models as shown in Fig. 1. The hard target of each training sample is either 1 or 0, depending on the class from which the sample is sampled, and the soft target is obtained from the posterior probability with which the sample belongs to the first class. The objective function is cross entropy, defined by

$$\mathcal{L}(w) = - \sum_i [t_i \ln\{y_i(w)\} + (1 - t_i) \ln\{1 - y_i(w)\}]$$

where t_i represents the target (either soft or hard).

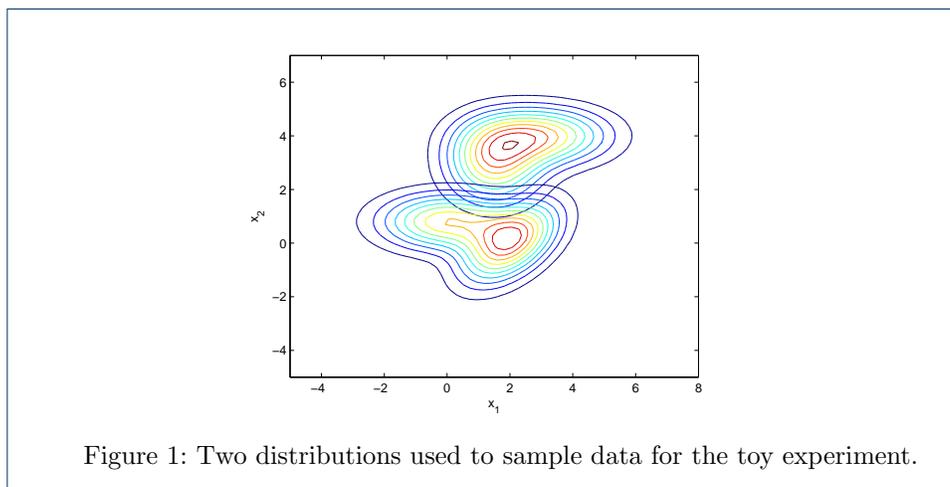
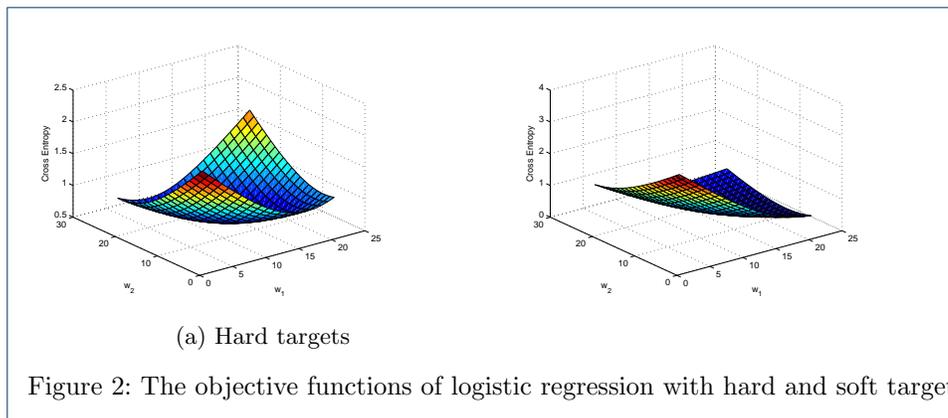


Fig. 2 shows the objective function $\mathcal{L}(w)$ with hard and soft targets. It can be seen that soft targets indeed lead to a smoother objective function, which makes it easier to train the model. Note that this is just a toy experiment. For deep models in practice, the objective functions are highly complex and bumpy. Nevertheless, it is still reasonable to expect that soft targets result in smoother objective functions. We argue that this smoothness is attributed to the less discriminative information involved in soft targets, and so less knowledge to learn in model training.

The smoothness of the objective function is highly desirable particularly in training deep and complex models such as RNNs. It can significantly mitigate the difficulties associated with the twisted objective function caused by layer-by-layer nonlinear composition. In this paper, we employ this property of dark knowledge transfer learning to train RNN models, which is hard and tricky with other methods.



3.3 Regularization view

It has been known that including both soft and hard targets improves performance with appropriate setting of a weight factor to balance their relative contributions [22]. This can be formulated as a regularized training problem, with the objective function given by:

$$\begin{aligned} \mathcal{L}(\theta) &= \alpha \mathcal{L}_H(\theta) + \mathcal{L}_S(\theta) \\ &= \sum_i \sum_j (\alpha t_{ij} + p_{ij}) \ln\{y_{ij}(\theta)\} \end{aligned}$$

where θ represents the parameters of the model, $\mathcal{L}_H(\theta)$ and $\mathcal{L}_S(\theta)$ are the cost associated with the hard and soft targets, respectively, and α is the weight factor. Additionally, t_{ij} and p_{ij} are the hard and soft targets for the i -th sample on the j -th class, respectively. Note that $\mathcal{L}_H(\theta)$ is the objective function of the conventional supervised training, and so $\mathcal{L}_S(\theta)$ plays a role of regularization. The effect of the regularization term is to force the model under training (child model) to mimic the teacher model, a way of knowledge transfer. In this study, a DNN model is used as the teacher model to regularize the training of an RNN. With this regularization, the RNN training looks for optima which produce similar targets as the DNN does, so the risk of over-fitting and under-fitting can be largely reduced.

3.4 Pre-training view

Instead of training the model with soft and hard targets altogether, we can first train a reasonable model with soft targets, and then refine the model with hard targets. By this way, the transfer learning plays the role of pre-training, and the conventional supervised training plays the role of fine-tuning. The rationale is that the smooth objective function associated with soft targets allows quicker and easier optimization and so can be used to conduct model initialization. However since the information involved in soft targets is less discriminative, refinement with hard targets tends to be helpful. This can be informally interpreted as teaching the model with less but important discriminative information firstly, and once the model is strong enough, more discriminative information can be learned.

This leads to a new pre-training strategy based on dark knowledge transfer. In the conventional pre-training approaches based on either restricted Boltzmann machines (RBM) [25] or auto-encoders (AE) [26], simple models are trained and stacked to construct complex models. The dark knowledge pre-training functions in a different way: it smoothes the objective function with less discriminative information (soft targets) so the model becomes trainable, though the model structure does not change. This approach possesses several advantages: (1) it is totally supervised and so more task-oriented; (2) it pre-trains the model as a whole, instead of layer by layer, so tends to be fast; (3) it can be used to pre-train any complex models for which the layer structure is not clear, such as the RNN model that we focus on in this paper.

The pre-training view is related to the curriculum training method discussed in [27], where training samples that are easy to learn are firstly selected to train the model, while more difficult ones are selected later when the model has been fairly strong. In the dark knowledge pre-training, the soft targets can be regarded as easy samples for pre-training, and hard targets as difficult samples for fine-tuning.

Interestingly, the regularization view and the pre-training view are closely related. The pre-training is essentially a regularization that places the model to some location in the parameter space where good local minima can be easily reached. This relationship between regularization and pre-training has been discussed in the context of DNN training [28].

4 Experiments

To verify the proposed method, we use it to train RNN acoustic models for an ASR task which is known to be difficult. The experiments are conducted on the Aurora4 database in noisy conditions, and the data profile is largely standard: 7137 utterances for model training, 4620 utterances for development and 4620 utterances for testing. The Kaldi toolkit^[1] is used to conduct the model training and performance evaluation, and the process largely follows the Aurora4 s5 recipe for GPU-based DNN training. Specifically, the training starts from constructing a system based on Gaussian mixture models (GMM) with the standard 13-dimensional MFCC features plus the first and second order derivatives. A DNN system is then trained with the alignment provided by the GMM system. The feature used for the DNN system is the 40-dimensional Fbanks. A symmetric 11-frame window is applied to concatenate neighboring frames, and an LDA transform is used to reduce the feature dimension to 200, which forms the DNN input. The DNN architecture involves 4 hidden layers and each layer consists of 2048 units. The output layer is composed of 2008 units, equal to the total number of Gaussian mixtures in the GMM system. The cross entropy is used as the training criterion, and the stochastic gradient descent (SGD) algorithm is employed to perform the training.

In the dark knowledge transfer learning, the trained DNN model is used as the teacher model to generate soft targets for the RNN training. The RNN is based on the LSTM structure, where the input features are the 40-dimensional Fbanks, and the output units correspond to the Gaussian mixtures as in the DNN. The

^[1]<http://kaldi.sourceforge.net/>

momentum is empirically set to 0.9, and the starting learning rate is set to 0.0001 by default.

The experimental results are reported in Table 1. The performance is evaluated in terms of two criteria: the frame accuracy (FA) and the word error rate (WER). While FA is more related to the training criterion (cross entropy), WER is more important for speech recognition. In Table 1, the FAs are reported on both the training set (TR FA) and the cross validation set (CV FA), and the WER is reported on the test set.

In Table 1, RNN-0 is the RNN baseline trained with hard targets. RNN-T1 and RNN-T2 are trained with dark knowledge transfer, where the temperature T is set to 1 and 2 respectively. For each dark knowledge transfer model, the soft targets are employed in three ways: in the ‘soft’ way, only soft targets are used in RNN training; in the ‘reg.’ way, the soft and hard targets are used together, and the soft targets play the role of regularization; and in the ‘pretrain’ way, the soft targets and the hard targets are used sequentially, and the soft targets play the role of pre-training. The weight factor in the regularization approach is empirically set to 0.5.

Table 1: Results with Different Models and Training Methods

	Targets	TR FA%	CV FA%	WER%
DNN	Hard	63.0	45.2	11.40
RNN-0	Hard	67.3	51.9	13.57
RNN-T1 (soft)	Soft	59.4	49.9	11.46
RNN-T1 (reg.)	Soft + Hard	67.5	53.7	10.84
RNN-T1 (pretrain)	Soft + Hard	65.5	54.2	10.71
RNN-T2 (soft)	Soft	58.2	49.5	11.32
RNN-T2 (reg.)	Soft + Hard	65.8	53.3	10.88
RNN-T2 (pretrain)	Soft + Hard	64.6	54.1	10.57

It can be observed that the RNN baseline (RNN-0) can not beat the DNN baseline in terms of WER, although much effort has been devoted to calibrate the training process, including various trials on different learning rates and momentum values. This is consistent with the results published with the Kaldi recipe. Note that this does not mean RNNs are inferior to DNNs. From the FA results, it is clear that the RNN model leads to better quality in terms of the training objective. Unfortunately, this advantage is not propagated to WER on the test set. Additionally, the results shown here can not be interpreted as that RNNs are not suitable for ASR (in terms of WER). In fact several researchers have reported better WERs with RNNs, e.g., [8, 9, 10, 12]. Our results just say that with the Aurora4 database, the RNN with the *basic* training method does not generalize well in terms of WER, although it works well in terms of the training criterion.

This problem can be largely solved by the dark knowledge transfer learning, as demonstrated by the results of the RNN-T1 and RNN-T2 systems. It can be seen that with the soft targets only, the RNN system obtains equal ($T=1$) or even better ($T=2$) performance in comparison with the DNN baseline, which means that the knowledge embedded in the DNN model has been transferred to the RNN model, and the knowledge can be arranged in a better form within the RNN structure. Paying attention to the FA results, it can be seen that the knowledge transfer learning does not improve accuracy on the training set, but leads to better or close

FAs on the CV set compared to the DNN and RNN baseline. This indicates that transfer learning with soft targets sacrifices the FA performance on the training set a little, but leads to better generalization on the CV set. Additionally, the advantage on WER indicates that the generalization is improved not only in the sense of data sets, but also in the sense of evaluation metrics.

When combining soft and hard targets, either in the way of regularization or pre-training, the performance in terms of both FA and WER is improved. This confirms the hypothesis that the knowledge transfer learning does play roles of regularization and pre-training. Note that in all these cases, the FA results on the training set are lower than that of the RNN baseline, which confirms that the advantage of the knowledge transform learning resides in improving generalizability of the resultant model. When comparing the two dark knowledge RNN systems with different temperatures T , we see $T=2$ leads to little worse FAs on the training and CV set, but slightly better WERs. This confirms that a higher temperature generates smoother objective functions and leads to better generalization.

5 Conclusion

We proposed a novel RNN training method based on dark knowledge transfer learning. The experimental results on the ASR task demonstrated that knowledge learned by simple models can be effectively used to guide training complex models. This knowledge can be used either as a regularization or for pre-training, and both approaches can lead to models that are more generalizable, a desired property for complex models. The future work involves applying this technique to more complex models that are difficult to train with conventional approaches, for example deep RNNs. Knowledge transfer between heterogeneous models is under investigation as well, e.g., between probabilistic models and neural models.

Acknowledgement

This research was supported by the National Science Foundation of China (NSFC) under the project No. 61371136, and the MESTDC PhD Foundation Project No. 20130002120011. It was also supported by Sinovoice and Huilan Ltd.

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